Mathematics in India

There is little known about the history of Indian mathematics; this is due to a small amount of authentic records containing their mathematics. The first known mathematics was preserved in the city Mohenjo Daro, during the time of the Indus Valley Civilization. The Indus Valley Civilization is thought to have been settled around 2,500 B.C.E. Mathematics was found everywhere in Mohenjo Daro, from its advanced architecture to its methods of measurement, counting and weighing items. The Indus Valley Civilization rivaled the other great ancient civilizations of its time in both knowledge and architecture styles. Examples of their architectural advancements were their tiled bathrooms, brick buildings, and temples, which all required a high level of geometrical understanding (Eves 181).

There is also evidence of a written numerical system imprinted on seals from the Indus Valley Civilization, consisting of the numbers one through thirteen depicted by vertical lines. After the findings at Mohenjo Daro there was little evidence of numbers being written down, but there is evidence of maths and numbers in the Vedas, specifically, the usage of the number eight in the *Rgveda*. These writings suggest that even though there is nothing directly stating the numbers, the people of the Indus Valley Civilization must have had a very sophisticated numerical system. This is in contrast to the Romans whose numerical system did not go farther than ten to the exponent of four, where the Indus Valley Civilization at the same time had knowledge of denominations as large as ten to the exponent of twelve, which is suggested by the *Yajurveda Samhita* (Singh 20).

After the Indus Valley Civilization disappeared the Aryan peoples started expanding into India. Indian mathematics can be split into two periods; the first of the two, coined the Sulvasutra (also written as Sulbasutra) period, which goes up until 200 C.E. The *Sulvasutras* are also texts that are appendices to the Vedas. The literal meaning of *Sulvasutra* is “the rules of the cord”; the texts written in this period are dated sometime between 800B.C.E. and 200 C.E. (Cajori 84). It was during this period the great grammarian Panini, who perfected the Sanskrit language and the Buddha became very influential. There are three different types of *ganita* (mathematics) found in ancient Buddhist texts; the first is finger arithmetic (*mudra)*, the second mental arithmetic (*ganana)* and the third higher arithmetic (*samkhyana)*(Singh 7).

It was also in this period that mathematics were taught and learned for the purpose of

geometry, to build temples and aid in other architecture. The *Sulvasutras* themselves were part of the Kalpasutras, and explained how to construct the sacrificial altars used in Hindu rituals. The *Sulvasutras* also contained some of the first references of the formula known around the world today as the Pythagorean Theorem. It is stated in the *Sulvasutras* the diagonal "…produces as much as is produced individually by the two sides", which shows they understood the idea of Pythagorean Theorem before it was ever proven as a theorem (Berlinghoff 139). Among the geometrical rules referring to the Pythagorean Theorem, there are references to the expression of the square root of two down to five decimals; others such as Heron the Elder in 100 B.C.E. also knew a similar method of approximation (Cajori 43).

One of the most famous rulers of the Mauryan Empire (King Asoka 272-232 B.C.E.) gives us an insight as to how early on the Hindu people were using the number system we use today. King Asoka built stone pillars in every major city in India, many of which still stand today. It is on these stone pillars we find the earliest examples of the Hindu-Arabic number system that is currently used. It is not only on these pillars that you can find written numbers, on the walls of a cave at the top of Nanaghat hill (near Poona) are numerous inscriptions of numerals. A more complete list of these numerals can be found in another cave, with these writings dated in the first or second century C.E. There are different theories as to where these symbols came from. Some would say they were from the Indus Valley Civilizations pictographic writing; another theory is that they have evolved from the Egyptians pictographs (Singh 26-28). Independently of where they came from, these depictions do not use the zero and decimal system that we now associate with Indian Mathematics (Eves 19). Even though these are some of the first depictions of our number systems there was no evidence to show the Hindus ever used any other number system (Singh 8).

There is little known about why the base ten system was used, but it is speculated it was due to how we count on our fingers. The Hindus were also one of the first to use a symbol to indicate a place value of zero; the Hindus used a small circle to indicate that the place value was empty. The mathematicians of India were not only one of the first to have a symbol for the missing place, they were also the first to explore zero as an actual quantity in itself. Thinking about numbers in this way was one major step above the mathematics of the ancient Greeks. It was thinking about numbers in this abstract way that enabled the Hindus to start doing math algebraically. Unfortunately, the usage of the base ten systems and zero as a number both took

centuries to be accepted in European mathematics. It was after this period Hindu Mathematics was able to really flourish (Berlinghoff 80).

As the Sulvasutra period came to an end, Indian Mathematics started to turn towards other practical uses. This period was called the astronomical and mathematical period, which dated from 400 C.E. to around 1200 C.E. (Cajori 84). This period was heavily influenced by outside forces; with India being invaded by other empires came outside knowledge of geometry, astrology and other mathematics. Unlike other countries that quit placing emphasis on investigating sciences while invasions took place, India turned the situation into an opportunity to learn from these new people. With this new knowledge the Indians placed more emphasis on learning which lead to founding universities. As a result India became a center for learning everything from the sciences to the arts. Mathematics had always been one of the most honored sciences, as suggested by the *Vedanga Jyotisa,* which states: "As the crests on the heads of peacocks, as the gems on the hoods of snakes, so is *ganita* at the top of the sciences known as the *Vedanha*." (Singh 7).

From then on mathematics was found in many different literary works such as the Puranas. The Puranas are literary works designed to spread education about historical and religious information among the peoples. Even the oldest of these works have references to place values and the base ten system; there are similar references in Patanjalis *Yoga-Sutra*. One of the first important astronomical works was written anonymously and is titled the *Surya Siddhanta*, which is translated as "Knowledge from the Sun". The *Surya Siddhanta* contained mathematics related to astronomical events but however, it did not have a specific section on mathematics. It did, however, have a more important role in influencing another great piece of literature, written a century later.

Varaha Mihira wrote the *Panca Siddhantika*, which contains a comprehensive summary of the trigonometry known by the early Hindus. An anonymous document written on birch bark was found in 1881 that had been buried since perhaps the eighth century. It is likely a copy of an older manuscript dated (from the style of verse) around the third or fourth century (Cajori 84-85). It contains methods of algebraic computation. This arithmetic is termed *patiganita,* coming from the words, *pati,* which means "board" and *ganita* meaning "science of calculation". Thus *patinganita,* is the science of calculation that requires it being written. However, sometimes the carrying out of arithmetic was called dust-work or *dhuli-karma* because they would write their

arithmetic in the sand instead of on a board.

After the *Panca Siddhantika* was written, the Hindu astronomer Aryabhata wrote his self entitled Siddhanta, which contained a whole chapter on mathematics. This chapter included one of the best estimations of the irrational number pi (π), the only closer estimation of the time had been made only fifty years prior by a Chinese scholar Zu Chongzhi. After the *Aryabhata*, it was common to include a chapter in astronomical texts specifically on the mathematics being used. Following Aryabhata mathematics continued to thrive in India, spurring on the work of Brahmangupta.

Brahmangupta's work the *Brahma-sphuta-siddhanta* ("Revised System of Brahma") contains two chapters on mathematics, and some of the first rules for negative numbers. Both Aryabhata and Brahmangupta could solve linear equations, with Brahmangupta taking it one step further to solve more difficult equations containing squares. He was also one of the first to work with negative quantities; regarding them as debts, he stated rules of addition, multiplication, subtraction and division of negatives. Even with these rules stated by a well-known scholar, people and mathematicians alike were still skeptical of these non-tangible numbers. It was later when Bhaskara II took Brahmangupta's ideas and generalized them, giving a method of solution of equations $nx^2+b=y^2$ (whenever a solution existed), as well as solutions with negative numbers (Berlinghoff 25-28, 93-94).

After Bhaskara II, there were few recognized mathematical works, but we do have the works of Sripati. Sripati wrote a *Ganita-sara*, which can be translated as the "Quintessence of Calculation", which helped refine the Hindu method of completing the square (Cajori 94). During this time period there were great advancements in geometry. Aryabhata's advancements in a method of approximating sines, led to his table of sines, which correspond to the particular angle. This was the beginning of the emphasis on estimation (Berlinghoff 186).

Indian mathematicians took the idea of approximation to another level, taking simple ideas and using them to develop sophisticated formulas to solve or approximate difficult solutions to problems. With an interest in algebra Indian mathematicians were able to compute square and cubed roots. They were also able to do the sums of arithmetic progression, this led to mathematics being investigated for its own sake, which you can see in how the problems were worded. They had essentially the same formula for the quadratic equation as we do today, with their version being expressed in words, as many of their formulas were. The problems in their

texts were often posed in a playful manner, an example from Bhaskara II, describes monkeys skipping through a grove and applying them to the mathematics at hand (Berlinghoff 27). Many of the mathematicians of India made discoveries of approximation by building upon one another; the formulas becoming more sophisticated as time goes on, it was these discoveries that anticipated ideas later rediscovered by European mathematicians.

Due to the location of India, in comparison to European countries, Indian mathematics almost always traveled to European countries through Arabic mathematicians. These Arabic mathematicians learned of astronomy, among other ideas as well, and took the Hindu trigonometry and expanded upon it. It is through this translation of ideas, that many of our mathematical terms are derived; for example "sine" comes from the Hindu *jya* (a cord for measurement) that the Arabs changed to *jiba*, which then came to be falsely interpreted as cove which is *sinus* in Latin, ultimately leading to the modern day "sine" (Berilinghoff 187).

Hindu mathematicians were the first to create many of the numbers and formulas we use today. It was their number system that allows us to do simple math efficiently and effectively, instead of the minute system used in the Roman Empire. The Hindus were advanced in their geometry, which enabled them to build elaborate temples and cities. There is also evidence of numbers and their place value system in the Vedas. This enabled the Brahmins (priestly class) to learn and explore mathematics. However, it was not only the Brahmins that were able to engage in mathematics, but also the Kshatriyas who took care of war and government matters. This led to the practical uses of mathematics for temple building, geometry, and most importantly astronomy and helped to pave the way for future generations.

References and Further Recommended Readings:

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Related Websites

http://www.hinduism.co.za/vedic.htm - History of Mathematics in India

http://en.wikipedia.org/wiki/Sulba_Sutras

http://www.archive.org/stream/arabhatiyawithc00arya@page/n3/mode/2up

http://www.math.tamu.edu/~dallen/history/1000bc/1000bc.html

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